

DETERMINATION OF CHARACTERISTIC VALUES IN NBR 7190: 1997 COMPARED WITH NONPARAMETRIC ESTIMATORS OF SAMPLE QUANTILES

DETERMINAÇÃO DO VALOR CARACTERÍSTICO NA NBR 7190: 1997 E ESTIMATIVAS NÃO PARAMÉTRICAS DE QUANTIS AMOSTRAIS

Carlos Alberto Oliveira de Matos ⁽¹⁾ (P)

(1) Prof. Dr., UNESP – Universidade Estadual Paulista “Júlio de Mesquita Filho”, Campus Experimental de Itapeva, Itapeva, Brasil.

Contact address: carlos@itapeva.unesp.br; (P) Presenter

Identification code: T3-18

Abstract

According to definition of NBR 7190: 1997, the characteristic value, unless specified otherwise, is the lower characteristic value $X_{k,inf}$ which is the value, smaller than mean value, which has only 5% probability of being reached in a given lot of material. In its Annex B, which has a normative character, it defines experimental methods for determination of sixteen wood properties for structural designs and the expression for estimation of the characteristic value.

The definition of characteristic value corresponds, in Statistics, to a quantile of 0.05 order, indicated by $q(0.05)$, such that 5% of observations are smaller than $q(0.05)$.

The present article compares the estimation of characteristic value defined in Brazilian standard with eight different definitions of nonparametric estimates sample quantiles most commonly used in statistical computing packages through simulation of pseudorandom variables using the Monte Carlo Method (MCM) in twelve sample sizes and five conditions of sample variability.

The results are analyzed according to desirable properties for a sample quantile estimator, to the adaptation to parameters of safety model of Brazilian standard and to increase the commercial use potential wood for structural design, using less restrictive estimates objectifying contribute to the ongoing discussion about the review and improvement of some aspects of the standard.

Keywords: characteristic value; Monte Carlo methods; Brazilian standard

Resumen

Segundo a definição da NBR 7190: 1997 o valor característico, salvo especificação em contrário, é o valor característico inferior $X_{k,inf}$, que é o valor, menor que o valor médio, que tem apenas 5% de probabilidade de ser atingido em um dado lote de material. No seu anexo B, que tem caráter normativo, ela define os métodos experimentais para a determinação de dezesseis propriedades das madeiras para projetos de estruturas e a expressão para a estimativa do valor característico.

A definição de valor característico corresponde, na Estatística, à medida do quantil de ordem 0,05, indicada por $q(0,05)$, tal que 5% das observações sejam menores do que $q(0,05)$.

O presente artigo compara a estimativa do valor característico definida na norma com oito diferentes definições de estimativas não paramétricas de quantis amostrais mais comumente usadas em pacotes de computação estatística através de simulação de variáveis pseudoaleatórias utilizando o método Monte Carlo (MMC) em doze diferentes condições de tamanho da amostra e de cinco condições de variabilidade amostral.

Os resultados são analisados em função das propriedades desejáveis para um estimador do quantil amostral, da adequação aos parâmetros do modelo de segurança da norma brasileira e do aumento do potencial de utilização comercial das madeiras para projetos de estruturas através da utilização de estimativas menos restritivas e objetivam contribuir para a discussão contínua acerca da revisão e do aperfeiçoamento de alguns aspectos da norma.

Palabras clave: valor característico; Método Monte Carlo; Norma Brasileira; NBR

1. BACKGROUND

The supply chain of construction has great social and economic importance in Brazil. It is responsible for 15.5% of GDP (Gross Domestic Product) and presents relevant social role in generating jobs, representing 6% of total employed persons, which characterizes the sector as the most employer in the country.

In Brazil, there is a widespread prejudice against the more intensive use of wood in construction of buildings because of the tradition inherited from the Portuguese colonizers, the ignorance of the properties of wood and insistence on outdated construction methods.

The Brazilian Association of Technical Standards (ABNT) is National Forum for Standardization published the NBR 7190:1997 - Design of wooden structures in 1997 replacing the NBR 7190:1982. "This standard sets the general conditions that must be followed in the design, implementation and control of the common structures of wood, such as bridges, roofs, floors and falsework." [1]

According to the definition of NBR 7190: 1997, the characteristic value, unless otherwise specified, is the smallest characteristic value $X_{k,low}$, which is the value, lower than the mean value, which has only 5% probability to be not achieved in a given batch of material.

The Annex B, which has a normative character, defines experimental methods for the determination of sixteen properties of wood for structural projects or that are used exclusively to compare resistance among different species:

- a) moisture content;
- b) density;
- c) compression parallel to grain;
- d) tension parallel to grain;
- e) compression perpendicular to grain;
- f) tension perpendicular to grain;
- g) shear;
- h) cracking;
- i) bending;
- j) hardness;
- k) impact resistance in flexion;
- l) embedding;
- m) shear strength at the bondline ;
- n) normal tension to the bondlinhe;
- o) resistance of toothed and beveled seams;

and presents the expression (1) for estimating characteristic values:

$$x_{wk} = \left(2 \frac{x_1 + x_2 + \dots + x_{\frac{n}{2}-1}}{\frac{n}{2}-1} - x_{\frac{n}{2}} \right) 1.1 \quad (1)$$

where results must be placed in ascending order $x_1 \leq x_2 \leq \dots \leq x_n$ disregarding the highest value when the number of specimens is odd. If x_{wk} is less than x_1 or less than seventy percent of the mean value, it is not considered valid.

The definition of characteristic value corresponds, in a statistical sense, to the 0.05 quantile. It is indicated by $q(0.05)$, and is a value such that 5% of the observations are smaller than $q(0.05)$.

Our objective is compare the estimator of the characteristic value defined in the standard with nine nonparametric estimators of sample quantiles most commonly used in statistical computing packages [2].

All sample quantiles are defined as weighted averages of consecutive order statistics [4]. Sample quantiles of type i are defined by (2):

$$Q[i](p) = (1 - \gamma)x[j] + \gamma x[j + 1] \quad (2)$$

where $1 \leq i \leq 9$, $(j - m)/n \leq p \leq (j - m + 1)/n$, $x[j]$ is the j th order statistic, n is the sample size, the value of γ is a function of $j = \text{floor}(np + m)$ and $np + m - j$, and m is a constant determined by the sample quantile type.

For types 1, 2 and 3, $Q[i](p)$ is a discontinuous function of p , with $m = 0$ when $i = 1$ and $i = 2$, and $m = -1/2$ when $i = 3$ [4].

Type 1 is the inverse of empirical distribution function where $\gamma = 0$ if $g = 0$, and 1 otherwise.

Type 2 is similar to type 1 but with averaging at discontinuities where $\gamma = 0.5$ if $g = 0$, and 1 otherwise.

Type 3 is a nearest even order statistic (SAS ® definition) where $\gamma = 0$ if $g = 0$, and 1 otherwise.

For types 4 through 9, $Q[i](p)$ is a continuous function of p , with $\gamma = g$ and m given below. The sample quantiles can be obtained equivalently by linear interpolation between the points $(p[k], x[k])$ where $x[k]$ is the k th order statistic. Specific expressions for $p[k]$ are given below [4].

Type 4 is a linear interpolation of the cumulative distribution function where $m = 0$ and $p[k] = k/n$.

Type 5 is a piecewise linear function where the knots are the values midway through the steps of the empirical cumulative distribution function where $m = 1/2$ and $p[k] = (k - 0.5)/n$. This is popular amongst hydrologists.

Type 6 is used by Minitab ® and by SPSS ®, where $m = p$ and $p[k] = k/n + 1$ and thus $p[k] = E[F(x[k])]$.

Type 7 is used by S ®, where $m = 1 - p$ and $p[k] = k - 1/n - 1$ and in this case $p[k] = \text{mode}[F(x[k])]$.

The resulting quantile estimates in Type 8 are approximately median-unbiased regardless of the distribution of x , where $m = (p + 1)/3$ and $p[k] = (k - 1/3)/(n + 1/3)$ and then $p[k] \cong \text{median}[F(x[k])]$.

The resulting quantile estimates in Type 9 are approximately unbiased for the expected order statistics if x is normally distributed where $m = p/4 + 3/8$ and $p[k] = (k - 3/8)/(n + 1/4)$.

2. METHODS

These comparisons were conducted through simulation of pseudo random variables using Monte Carlo methods. The study simulated eleven different sample sizes ($n = 6, 7, 8, 9, 10, 11, 12, 18, 24, 30, 36$) together with five variability conditions (coefficient of variation = 5, 10, 15, 20, 25). Fifty-five samples resulting from combinations of all levels of the two factors were generated 20,000 times. All samples were generated with normal distribution.

3. RESULTS

All resulting values were standardized by dividing by the estimate defined by the standard. This variable (z) is shown in Figure 1, which shows that this characteristic value estimator overestimates the estimates obtained with the other nine non-parametric estimators.

In most situations, the estimator used in the standard is about 9% higher and this result is 35% in the most extreme situation.

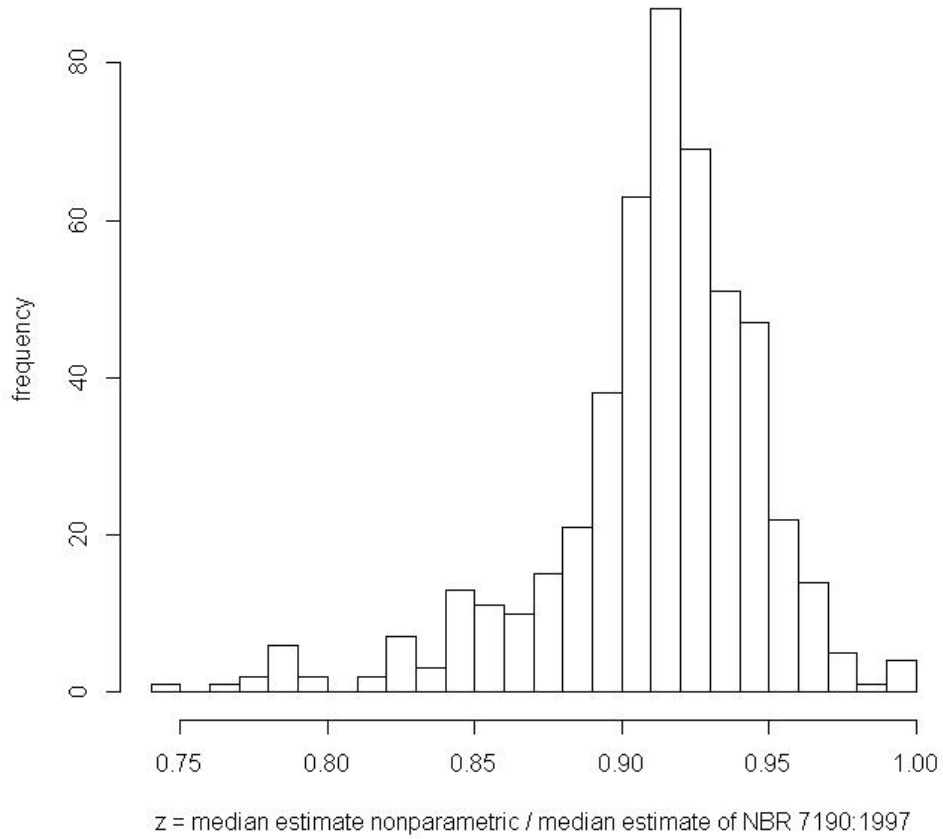


Figure 1: Histogram of z (median estimate nonparametric / median estimate of NBR 7190: 1997) in all 55 simulated situations.

The Figure 2 shows the distribution of the variable z to each of the estimators. The difference between the definition of Brazilian standard and nine estimators of sample quantiles becomes more visible.

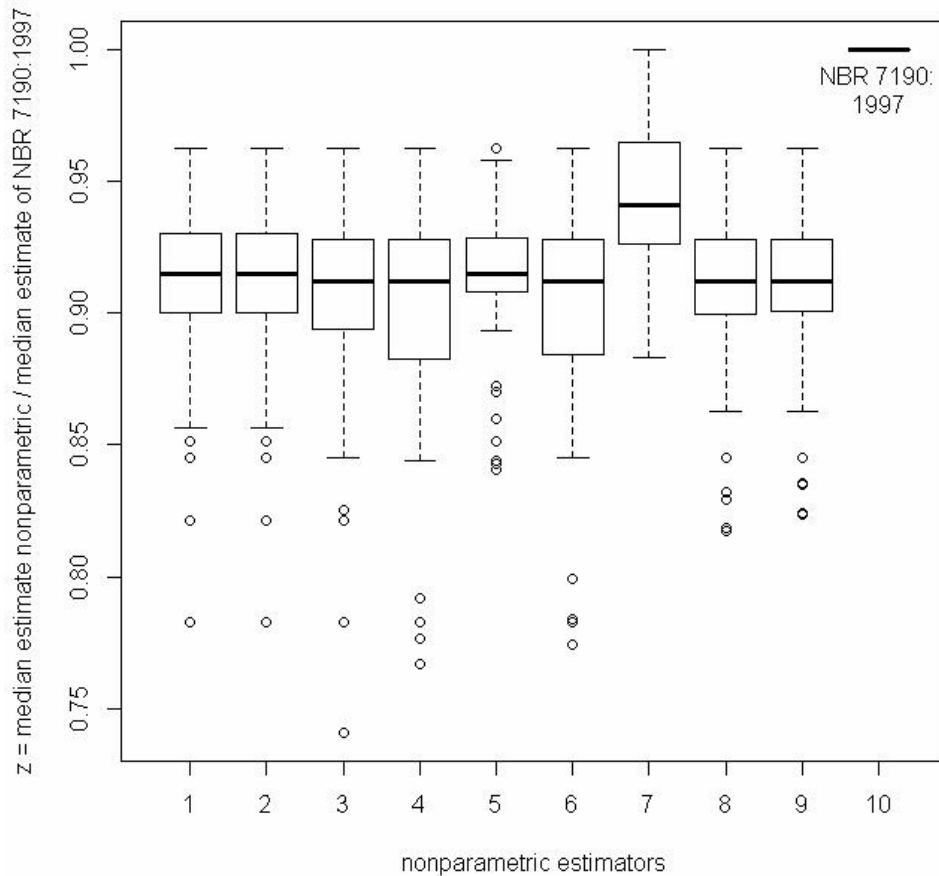


Figure 2: Comparison of the characteristic value defined in NBR 7190:1997 with nine nonparametric estimators of sample quantiles most commonly used in statistical computing packages.

This difference is smaller in relation to the estimator 7 which considers “the modal position $p_k = \text{mode } F(X_{(k)}) = (k-1)/(n-1)$. One nice property is that the vertices of $\hat{Q}_7(p)$ divide the range into $n-1$ intervals, and exactly $100p\%$ of the intervals lie to the left of $\hat{Q}_7(p)$ and $100(1-p)\%$ of the intervals lie to the right of $\hat{Q}_7(p)$ ” [3]. Unfortunately, this definition has only four of the six desirable properties for a sample quantile.

4. CONCLUSIONS

The results aims to contribute to the ongoing discussion on the revision and improvement of some aspects of this standard and are analyzed in terms of the following factors: desirable properties of a sample quantiles estimator, adequacy to the safety model parameters from the Brazilian standard and increased potential for commercial use of wood for structural projects through the use of less restrictive estimates.

REFERENCES

- [1] ABNT: NBR 7190 - Projeto de estruturas de Madeira. ABNT Rio de Janeiro, 1997.
- [2] R. J. Hyndman and Y. Fan: Sample Quantiles in Statistical Packages. *The American Statistician*, 50(4):361-365, 1996.
- [3] E. J. Gumbel: La Probabilité des Hypotheses. *Comptes Rendus de l'Academie des Sciences*, 209:645-647, 1939.
- [4] R Core Team. Sample Quantiles. In: *R Documentation*. Retrieved from <http://127.0.0.1:12440/library/stats/html/quantile.html>