

ESTIMATING THE FUNDAMENTAL PHYSICAL AND MECHANICAL PROPERTIES OF SOFTWOOD WITH AN INTEGRAL CALCULATION METHOD

ESTIMACIÓN DE LAS PROPIEDADES FÍSICAS Y MECÁNICAS FUNDAMENTALES DE MADERA DE CONÍFERAS CON UN MÉTODO DE CÁLCULO INTEGRAL

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Abstract

An integral calculation method (ICM) for estimating the fundamental physical and mechanical properties of softwood is proposed in this investigation. Unlike to most numerical models of wood, which are based on predicting wood properties based on the modelling of specific lumber pieces, the proposed method is based on the modelling of logs of softwood. Specifically, for the mechanical and physical properties of timber to be predicted, the method needs physical measurements of the log. The ICM consists of two basic calculation phases. In the first phase, the fibre orientation and the corresponding material properties of the log are analytically calculated for the entire log. In the second step, this data is imported into a minimalistic finite element analysis (FEA), which serves for predicting the physical and mechanical response of either the entire log or the corresponding sawn lumber. The method has initially been validated against 32 lumber boards in bending which resulted in an absolute numerical error of 10.2% on failure prediction. The calculation speed and robustness of the method was very high in comparison to similar models. It is expected that the ICM can contribute not only to lumber grading and research, but also to optimize the industrial logging and sawing processes. Additional experiments should be conducted in the future for tailoring the method to specific wood species and eventually transfer it to the industry.

Keywords: wood calculation method; material model; numerical model; physical and mechanical properties; wood anatomy

Resumen

Esta investigación propone un método de cálculo integral (ICM) para estimar las propiedades físicas y mecánicas fundamentales de la madera de coníferas. A diferencia de la mayoría de modelos numéricos de la madera, los cuales están basados en predecir las propiedades de la madera en base a modelizar piezas de madera específicas, el método propuesto se basa en la modelización de troncos de conífera. Específicamente, para que el modelo pueda predecir las propiedades de la madera, se precisan ciertas mediciones del tronco. El ICM consiste en dos fases de cálculo. En la primera fase, la dirección de la fibra y las propiedades correspondientes a esta desviación son calculadas para la totalidad del tronco. En la segunda fase, se realiza un análisis de elementos finitos (FEA) minimalista, lo que sirve para predecir la respuesta física y mecánica del tronco entero o cualquier pieza que pueda ser aserrada del mismo. El modelo se ha validado inicialmente frente a la carga de rotura de 32 vigas estructurales en flexión resultando en un error en la predicción de falla del 10.2%. La velocidad de cálculo y la robustez del método fue muy satisfactoria en relación a los modelos actuales. Se espera que el ICM pueda contribuir no solo a la clasificación e investigación de la madera sino también a la optimización de los procesos de corte. Se deberían realizar experimentos adicionales en el futuro para adaptar el método a las distintas especies y eventualmente transferirlo a la industria.

Palabras clave: método de cálculo de la madera; modelo material; modelo numérico; propiedades físicas y mecánicas; anatomía de la madera

1. INTRODUCTION

During the last decades, multiple stochastic [e.g. 1-3] and deterministic models [e.g. 2-11] have been proposed to estimate the engineering properties of timber. Fully stochastic models typically offer poor correlation of wood properties because of the large number of material or anatomical variables. Another drawback of pure statistic models is that they cannot provide an insight into the underlying physical principles which lead to the observed engineering properties, and thus it is complicated to improve our understanding of the wood material with those methods. On the other hand, pure deterministic models of wood are often too complex and its practical application outside of the research realm is in general unfeasible. Furthermore, the radial or cylindrical nature of the wood mechanical and physical properties, as well as the original bearing function of the timber in the tree is commonly disregarded in deterministic models.

This paper presents a new approach, that we termed integral calculation method (ICM), for calculating the physical and mechanical properties of softwood. The ICM is essentially deterministic, although it is relatively straightforward to include in the model statistical elements such as the measured variability of defect free wood. Unlike to previous lumber models, the presented ICM has two major novelties:

- First, it is not based on modelling lumber pieces, but on the modelling of the entire log of the softwood. This aspect drastically facilitates accounting for the cylindrical nature of the wood, and in general, considering the real the wood anatomy.
- Second, most of the calculations are performed analytically, and only a final calculation is conducted with the finite element method (FEM). This means that the ICM is mostly based on the calculation of simple algebraic equations, and not on the approximation of partial differential equations by numerical methods. Thus, the ICM can offer much higher calculation speed in comparison to other deterministic models, which can be seen as a critical factor prior to any practical application.

This publication presents thus the main aspects of such proposed ICM. For a more detailed description of the method, the reader is referred to a recent publication [10]. The ICM can be summarized in two main phases: (i) on the first phase, most material properties are calculated for the entire softwood log, (ii) on the second phase, such analytically calculated properties are used in a FE model to predict the engineering properties of the lumber. Both phases are detailed in the following.

2. PHASE ONE: ANALYTICAL CALCULATION OF THE LOG

The first phase of the calculation method consists of the calculation of the properties of the entire softwood log. Such computation is performed by calculating first the fiber orientation of the log, as most engineering properties depend on the fiber orientation. For this calculation to be performed, it is necessary to define first a suite of model input parameters involving the main physical features of the log – see an illustration of the necessary parameters in the Figure 1.

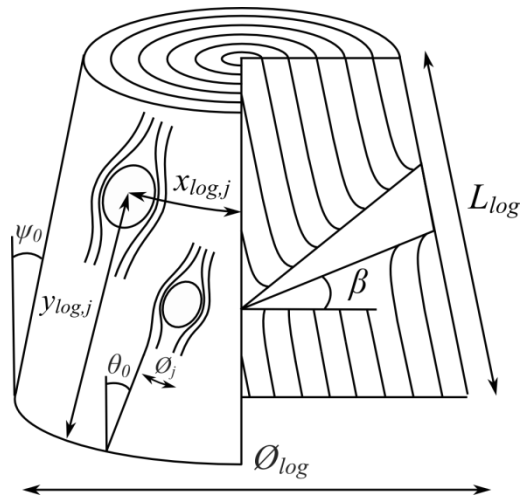


Figure 1: Main model input parameters, based on [10]

Thus, the input parameters that must be defined in the model include the diameter (\varnothing_{log}), length (L_{log}) and taper angle (ψ_0) of the log, the branch angle (β), the global fiber orientation angle or spiral grain angle (θ_0), and the diameter (\varnothing_j) and position of the knots ($x_{log,j}$ and $y_{log,j}$). Note that ψ_0 can simply be obtained out of the difference of \varnothing_{log} at the base and top of the log, and β can be estimated according to the height of the branch in the tree and the wood species. Furthermore, a measurement of these modelling inputs can be facilitated by current industrial equipment such as laser or x-ray technology.

In addition to those input parameters, it is also necessary to define the resolution of the calculation mesh - the material properties of the wood are to be computed in a grid of 3D points, and therefore it is necessary to define the size or spacing of that mesh. In principle, the ICM can be applied from meso- (few millimetres) to macro- or structural scale (several meters) because the mathematical principles are exactly the same, except that the size of the mesh is different.

Once the input parameters are defined, 6 operations are conducted, see an illustration in the Figure 2. These 6 operations are actually the core of the first phase of the proposed ICM, and involve the computation of about 38 algebraic and 2 simple differential equations that can be solved within a very short time lapse in a normal computer. The main contents of each of these 6 operations is in the following presented:

1. Slicing of the log. First, the volume of the trunk is sliced into a series of conical surfaces or rings ($S_{c,k}$). The material properties of the wood are to be calculated only for the points contained on those surfaces, and interpolated for the remaining regions. Note that the size of the mesh dictates the separation in between those rings, and depending on that size, those rings may represent the material (coarse mesh), the annual (fine mesh) or the seasonal (very fine mesh) growth rings of the tree. Such discretization of the trunk on several rings seems logical for calculating the wood properties, as it corresponds to the way the wood tissues grow in nature. As illustrated in the Figure 2, the internal rings (juvenile wood) are likely to be conical surfaces, while the external rings are likely to be truncated-conical surfaces. The position of the knots in each of that surfaces can easily be determined according to their external position ($x_{log,j}$ and $y_{log,j}$) and the branch angle (β).

2. Unwrapping of the conical rings. This operation mainly consists of transforming the 3D rings calculated in the previous step to 2D circular planes by unwrapping the $S_{c,k}$ surfaces. Note that unwrapping conical 3D rings results into circular sectors in 2D, meanwhile unwrapping the truncated conical surfaces results into portions of a circular sector, as the one illustrated in Figure 2. Such

unwrapping from 3D to 2D is very useful because the fiber orientation can much faster be calculated in 2D. The coordinates of the knots can now for each surface be defined on a 2D plane.

3. Conformal mapping. The previous circular sectors, and corresponding knots are transformed into rectangular planes ($S_{r,k}$) via conformal mapping. This means that the circular sectors previously considered (that represented the rings of the tree) can be transformed into rectangular planes preserving the local angles using a mapping function. This transformation is a very useful and powerful concept because it makes possible to calculate the fiber orientation in rectangular planes and this is much easier than in circular planes. The conformal mapping can straightforward be calculated by mapping the relative polar coordinates into relative Cartesian coordinates.

4. Computation of the fibre orientation. This step computes the fibre orientation for each of the rectangular planes of the previous step. There are 3 slopes to compute:

a. The first one is the tangential angle ($\theta_{r,k}$). This is the angle of the fibers that can be seen in the external surface of the log. In the ICM, this angle is calculated by the well-known theory of the flow-grain analogy [11]. This theory consists of calculating the wood fibers as the trajectory of a laminar fluid. However, in the ICM a small modification is done to the original theory. The trajectory is not directly calculated with a single analytical equation, but the complex potential function of the virtual fluid is defined, and the Milne-Thomson's circle theorem is applied instead. This modification is very advantageous from the computational point of view, because it permits to calculate the trajectory of the fluid as a single step in case of multiple knots, i.e. a single calculation step suffices for calculating the tangential fiber orientation for all the knots of each ring.

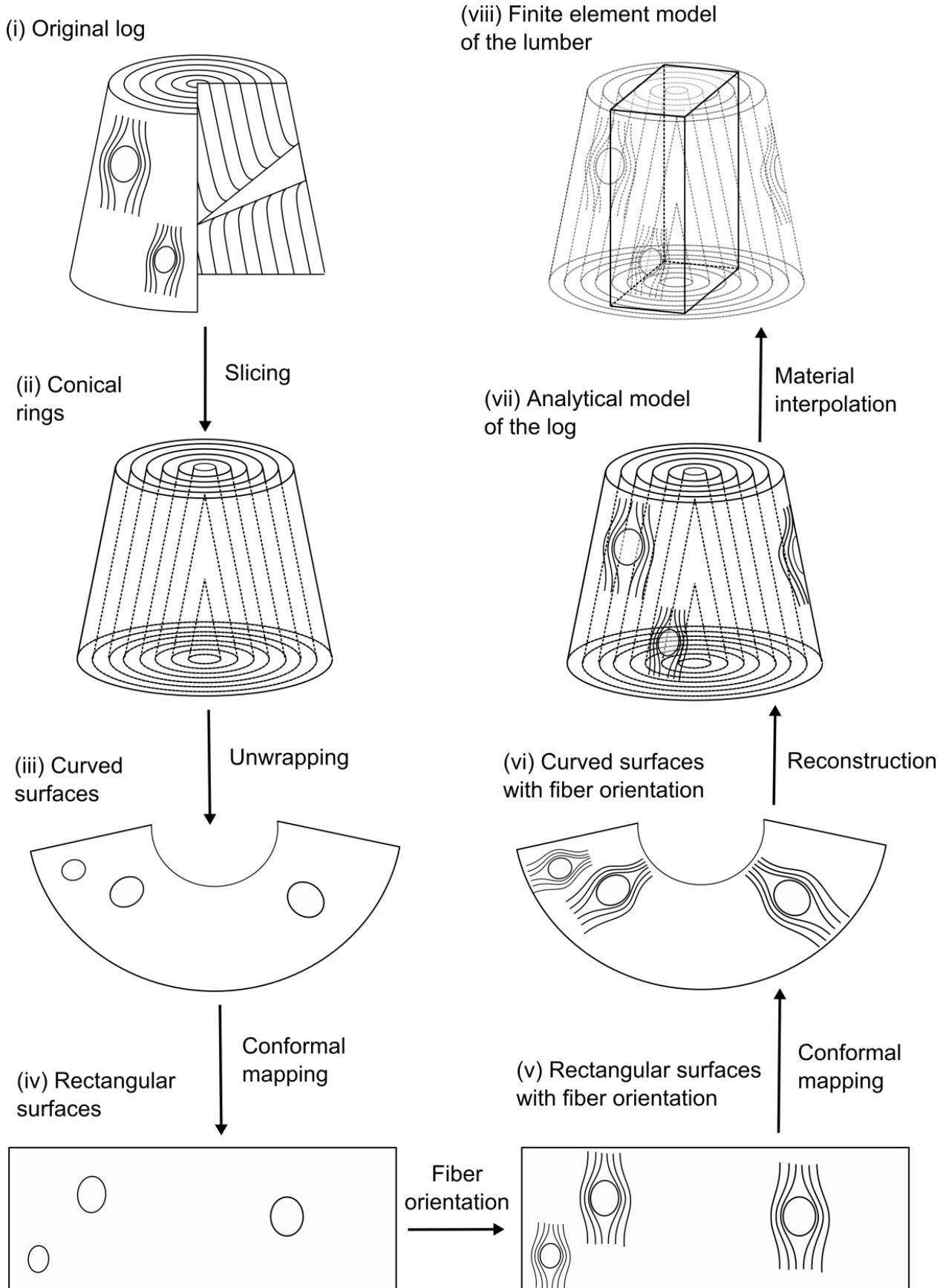


Figure 2: Main model operations, based on [10]

b. The second is the radial angle ($\psi_{r,k}$), also termed as dive angle. This angle is not visible before sawing, and consists of the slope of the fibers in the LR plane. In the ICM, the $\psi_{r,k}$ is calculated by fitting polynomial equations to experimental measurements, as it has been done in previous investigations [6,7]. In this case it has been found, that for the species *Pinus sylvestris* K., the dive angle distribution approximately matches an exponential function above the knots, and a logarithmic function below the knots. Such polynomial equations are applied for each point of the rectangular plane obtained in the previous step. In case that several knots are influencing to the same point, two different cases were established. When the knots are influencing the dive angle in the same direction, the dive angle was calculated as the maximum of the diving angle generated for each separate knot. Conversely, when knots are generating slopes in opposite directions, then the dive angle was taken as the summation of each single knot.

c. The third is the cylindrical angle ($\varphi_{c,k}$). Unlike to the previous slopes, this angle does not define the slope of the fibers, i.e. the L direction in each point of the log, but the exact position of the R and T axes. Such $\varphi_{c,k}$ angle is omitted in most of the deterministic models for wood. Considering for this angle in the ICM is however straightforward, because rectangular planes actually represent the rings of the log, and therefore the cylindrical angle can simply be calculated according to the horizontal relative coordinate of each point, see step v of Figure 2.

5. Reverse confocal mapping. Once the 3 angles ($\theta_{r,k}$, $\psi_{r,k}$ and $\varphi_{c,k}$) are calculated for each point of the rectangular planes ($S_{r,k}$), they can be mapped back to the circular planes ($S_{c,k}$) by using the inverse conformal mapping function defined on the step 3, which guaranties the preservation of those 3 local angles. This results on the complex fiber orientation of the circular surfaces out of the simpler calculation on the rectangular planes.

6. Reconstruction. The conical surfaces are wrapped back into the three-dimensional space, resulting on the original rings sliced on the step 1 with their corresponding fibre orientation angles. This means that the fiber orientation angles are now determined in the 3D space for each of the rings of the log, see an illustration of the result on the Figure 3. Once the angles are defined in the 3D space, they can be used to define the mechanical and physical properties of the wood according to continuum mechanics principles. More specifically, the three angles defining the orientation of the wood fibers are used for constructing a Tait-Bryan intrinsic rotation matrix (Q_k), such that:

$$Q_k = Q_{\vartheta,k} \cdot Q_{\theta,k} \cdot Q_{\psi,k} \quad (1)$$

where:

$$Q_{\vartheta,k} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \vartheta_{c,k} & -\sin \vartheta_{c,k} \\ 0 & \sin \vartheta_{c,k} & \cos \vartheta_{c,k} \end{pmatrix} \quad (2)$$

$$Q_{\theta,k} = \begin{pmatrix} \cos \theta_{c,k} & 0 & \sin \theta_{c,k} \\ 0 & 1 & 0 \\ -\sin \theta_{c,k} & 0 & \cos \theta_{c,k} \end{pmatrix} \quad (3)$$

$$Q_{\psi,k} = \begin{pmatrix} \cos \psi_{c,k} & -\sin \psi_{c,k} & 0 \\ \sin \psi_{c,k} & \cos \psi_{c,k} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4)$$

So that any property depending on the fiber direction can be calculated for each ring as:

$$K_k = Q_k^T \cdot K \cdot Q_k \quad (5)$$

where

$$K = \begin{pmatrix} k_L & 0 & 0 \\ 0 & k_R & 0 \\ 0 & 0 & k_T \end{pmatrix} \quad (6)$$

being k_L , k_R , and k_T are the wood properties in the L, R and T directions, respectively - the physical properties of wood are typically dependent only on the fiber direction and can be calculated according to the Eq. 5.

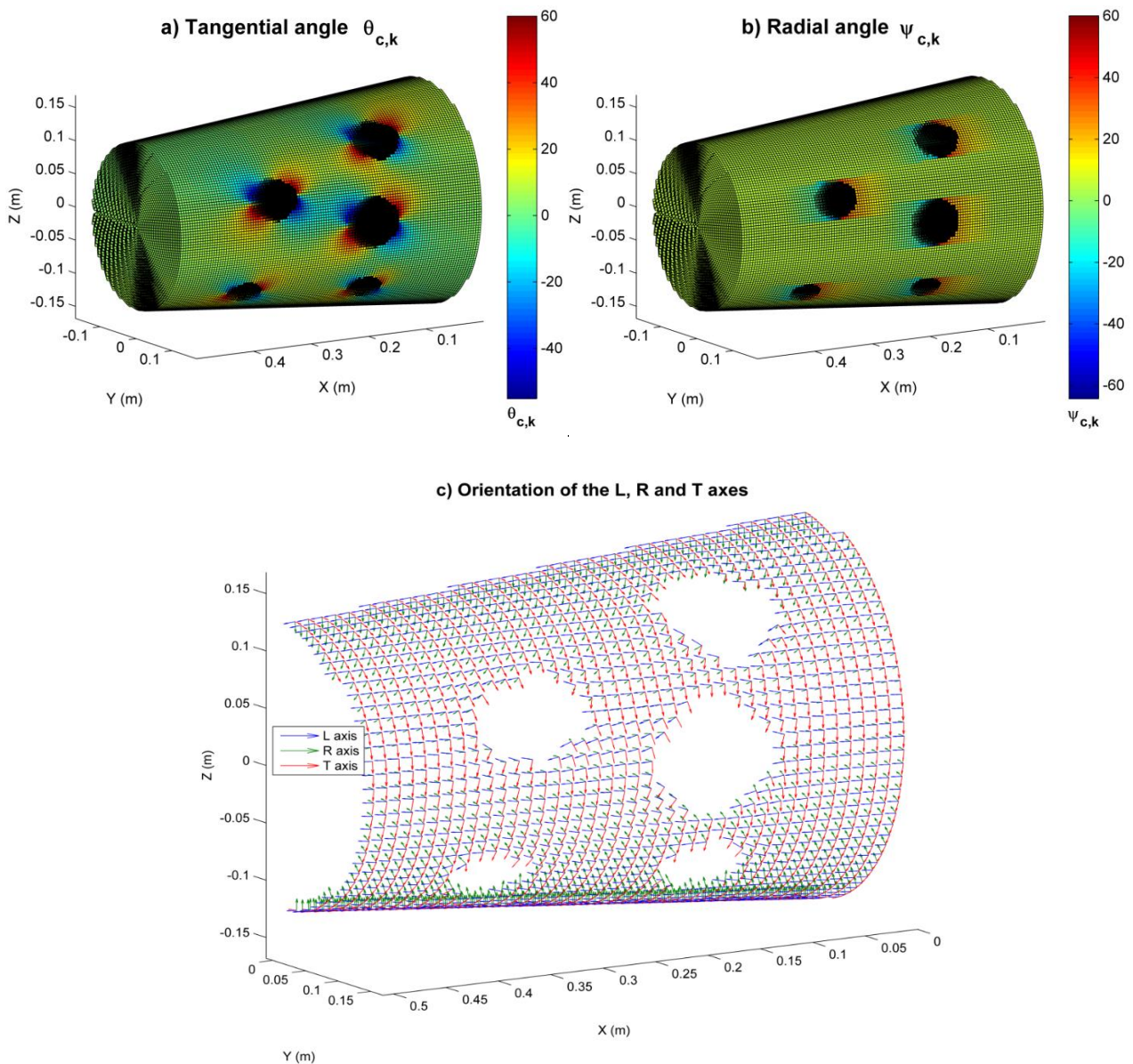


Figure 3: Illustration of (a) tangential angle, (b) radial angle and (c) orientation of material axes predicted by the model for a small log of 300mm in diameter and 500mm in length, based on [10]

On the other hand, the properties depending on both the fiber direction and the material plane, such as the elastic properties, can be calculated for each ring as

$$\mathbf{S}_k = \mathbf{J}_{\epsilon,k} \cdot \mathbf{S} \cdot \mathbf{J}_{\sigma,k}^{-1} \quad (7)$$

where \mathbf{S} is the compliance matrix

$$\mathbf{S} = \begin{pmatrix} \frac{1}{E_L} & \frac{-\nu_{LR}}{E_L} & \frac{-\nu_{LT}}{E_L} & 0 & 0 & 0 \\ \frac{-\nu_{LR}}{E_L} & \frac{1}{E_R} & \frac{-\nu_{TR}}{E_R} & 0 & 0 & 0 \\ \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{TR}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} \end{pmatrix} \quad (8)$$

and $\mathbf{J}_{\epsilon,k}$ and $\mathbf{J}_{\sigma,k}$ are the strain and stiffness transformation matrices, respectively.

Thus, after conducting the 6 operations described above according to the simple physical measurements of the log described in the Figure 1, it is possible to obtain an approximation of the basic physical \mathbf{K}_k and mechanical \mathbf{S}_k properties of the entire softwood log. Note that by including a randomly normal distributed contribution in the Eq. 6 and 8 according to the variability of defect-free wood properties, it would be possible to include the variability of the clear wood in the ICM. For instance, the Eq. 6 may be redefined such that

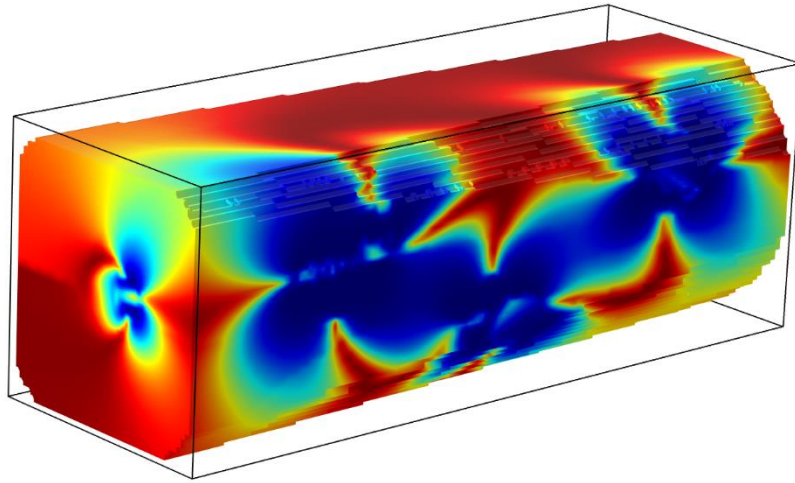
$$\mathbf{K} = \begin{pmatrix} k_L(\mu_L, s_L) & 0 & 0 \\ 0 & k_R(\mu_R, s_R) & 0 \\ 0 & 0 & k_T(\mu_T, s_T) \end{pmatrix} \quad (9)$$

where each directional property k is now obtained as a random number of a normal distribution with the measured average μ and variance s^2 .

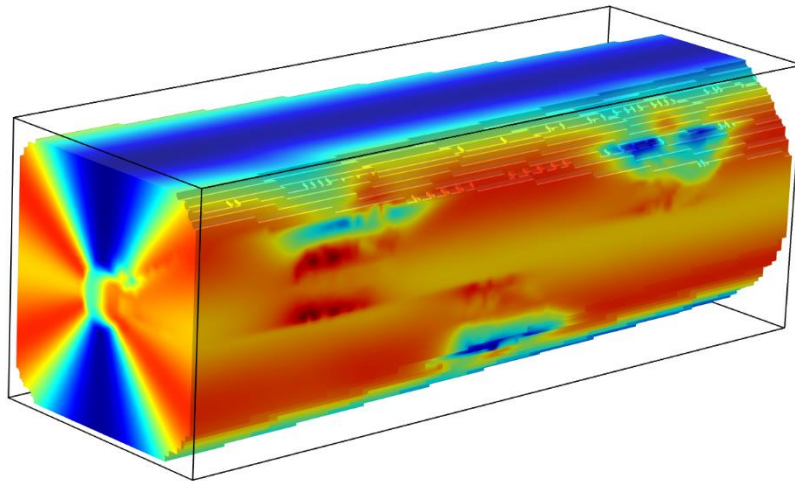
3. STEP TWO: FINITE ELEMENT MODEL OF THE WOOD

This step consists of conducting a finite element (FE) analysis that accounts for exactly the same global coordinate system of the step 1, so that the material properties of any wood piece which may be sawn from the original log can be interpolated from the previous analytical calculation (see an illustration on the last operation of Figure 2). In other words, by locating any lumber piece in the log, it is now possible to use the previously calculated material properties to conduct a numerical analysis. This FE computation may serve for either conducting a detailed physical and mechanical analysis, or calculating the effective (average) properties of the wood, e.g. global stiffness, moisture diffusion or thermal conductivity. See an illustration of FE results on the Figures 4 and 5.

(a)



(b)



(c)

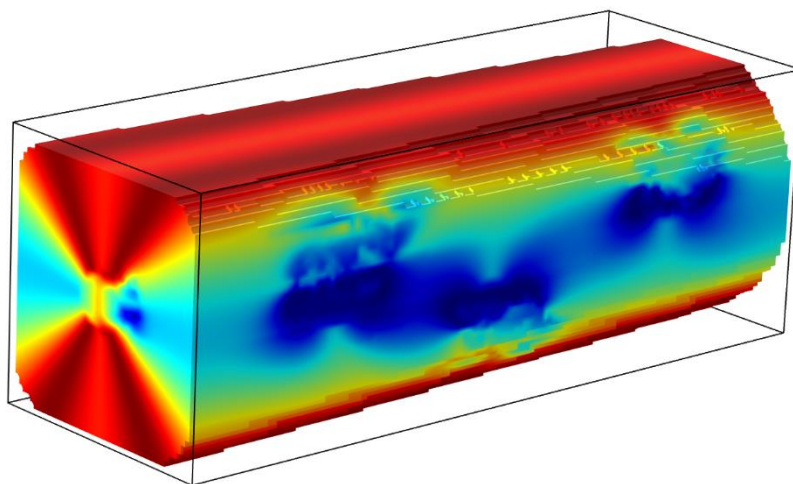
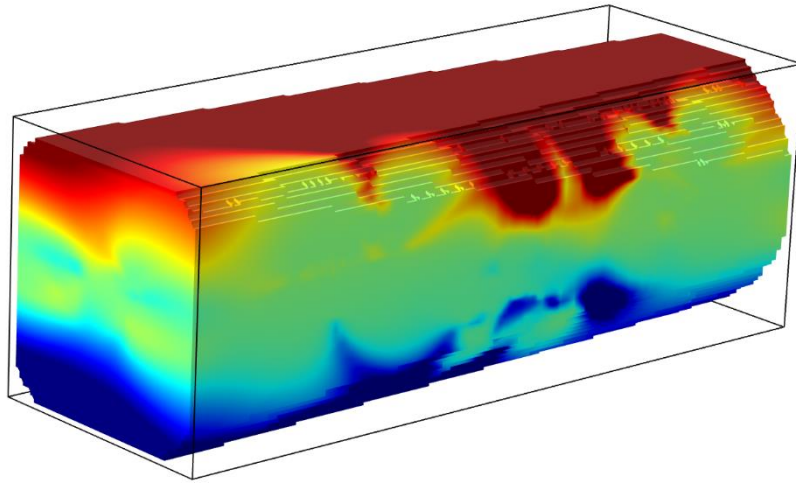
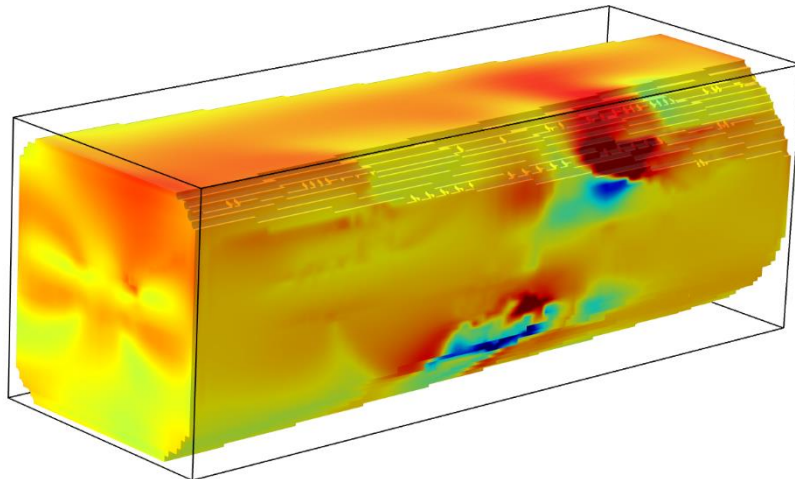


Figure 4: Illustration of Young moduli in the (a) longitudinal, (b) width and (c) depth beam directions used for the FE computations of a small lumber piece containing 3 knots. For simplicity hot colors indicate here large stiffness.

(a)



(b)



(c)

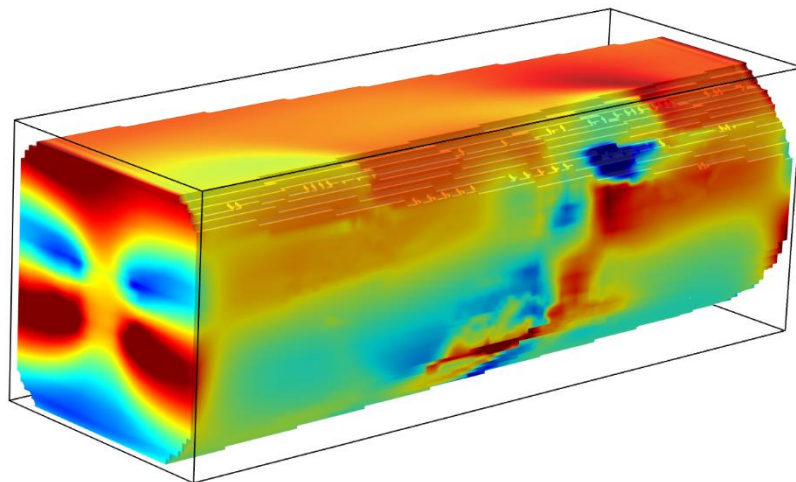


Figure 5: Illustration of the computed stresses in the (a) longitudinal, (b) width and (c) depth beam directions for a small lumber piece containing 3 knots. For simplicity hot colors indicate here large stress.

Note however that such FE analysis only involves the interpolation of the previous analytical properties, the application of the boundary conditions and the computation the solution of the problem. This dramatically increases the robustness and computational speed of this method in comparison to similar numerical models for the wood, i.e. FE calculations can be done in a very few seconds. Also, as the model serves for calculating the physical or mechanical response of any piece which might be sawn from the original log, it may allow not only for modelling the lumber but for optimizing the logging and the sawing industrial processes.

4. INITIAL VALIDATION

The model was initially validated against the failure load of 32 structural beams of *Pinus sylvestris* L. which were tested according to the 4-point bending standard procedure of EN 408. The numerical failure load was predicted with the Hoffman criterion via mean stress approach of 7mm in size, resulting in an absolute error of 10.2%. The initial results are encouraging and a more exhaustive experimental campaign that includes real wood measurements of full softwood logs should be performed. The model also should be tailored to one specific species to assess its potential for industrialization.

5. CONCLUSION

This research proposes a calculation method for the numerical modelling and estimation of the physical and mechanical properties of softwood. The calculation method has the particularity that it is based on the physical characteristics of the logs, rather than specific lumber pieces. This let us to account for conical and cylindrical nature of the timber, and therefore the fibre orientation can be calculated with higher accuracy. An additional advantage is that it can calculate any lumber piece which may be sawn from the log. Thus, the model should be useful not only for lumber grading, but for industrial logging and sawing.

Because most of the calculations are conducted analytically and only a final simplistic FE analysis is performed, this approach increases drastically the computational speed and robustness in comparison to previous numerical models. The initial validation with experiments has shown very promising results. The proposed calculation method is therefore expected to be useful in multiple applications yielding to a more efficient utilization of this material in the future. A more exhaustive validation that includes full field fiber measurements in softwood logs should be performed before assessing industrial implementation.

In addition, there are a number of aspects that still need to be addressed. The presented model considered one of the simplest cases, i.e. a straight log with rectilinear pith and circular section. However, it is obvious that more difficult geometrical log shapes can be found, such as curvilinear pith or elliptical cross sectional logs. Those aspects should be addressed in the future by implementing more sophisticated mapping approaches. Note however, that those issues are very difficult, or rather impossible to consider in most current numerical models of timber. This investigation is thus expected to contribute with a new approach for modelling the timber, which is not based on the anatomical modelling of lumber pieces, but on the modelling of the log itself. Also it is expected to contribute that many calculations of the fiber orientation can be performed analytically before numerical methods such as the finite element method are applied, and this drastically can increase the efficiency and speed of the calculations, which may be an essential aspect for the application of deterministic models of wood in the practice.

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